

## Probability

1. Five aircraft take part in a race from England to Australia. It is estimated that the chances, for the individual aircraft, of breaking the record are  $\frac{1}{4}$ ,  $\frac{2}{7}$ ,  $\frac{1}{3}$ ,  $\frac{2}{5}$  and  $\frac{1}{2}$  respectively. What is the probability that a new record will be set up? (Ans.  $\frac{25}{28}$ )
2. Two persons A and B are playing a game in which A's skill is twice B's. What is the chance that A will win at least 3 such games in a set of five? Under the same conditions, what is the chance that A will win three games before B wins two? (Ans.  $\frac{192}{243}$ ,  $\frac{48}{81}$ )
3. A die is thrown 5 times. What is the probability that
  - (a) 2 appears every time
  - (b) 2 appears exactly 4 times
  - (c) 2 appears at least 3 times.(Ans. (a)  $\frac{1}{7776}$ , (b)  $\frac{25}{7776}$ , (c)  $\frac{276}{7776}$ )
4. If a bag contains n balls and any number of balls are taken out at random, what is the probability that an even number of balls are taken? (Ans.  $\frac{2^{n-1}-1}{2^n-1}$ )
5. How many times a die should be thrown so that the probability of obtaining 1 is just greater than  $\frac{1}{2}$ ? (Ans. 4)
6. A and B throw a die alternately. If A begins and the throwing is continued until either A or B obtains 1. What is the probability that (a) A obtains 1 first (b) B obtains 1 first? (Ans.  $\frac{6}{11}$ ,  $\frac{5}{11}$ )
7. Two person A and B are to draw alternately one ball at a time from a bag containing 3 white and 2 black balls, the balls drawn not being replaced. If A begins, what chance has each of being the first to draw a white ball? If the balls are replaced as they are drawn, what are the respective chances of A and B? (Ans. A:  $\frac{7}{10}$ , B:  $\frac{3}{10}$ ; A:  $\frac{5}{7}$ , B:  $\frac{2}{7}$ )
8. A urn A contains 10 balls, 3 of which are white and a second urn B contains 12 balls, 4 of which are white. If one urn is drawn at random and a ball drawn from it, what is the chance that the ball is white? (Ans.  $\frac{19}{60}$ )
9. Two dices are thrown. Find the probability that
  - (a) the sum is at least 9;
  - (b) the difference of the numbers on the two dice is 1 or 2.(Ans.  $\frac{5}{18}$ ,  $\frac{1}{2}$ )
10. A die is tossed three times. What is the probability that the first toss will show odd, the second toss even, and the third a six? (Ans.  $\frac{1}{24}$ )

11. An urn contains a large number of black and white marbles in equal proportions and thoroughly mixed. A sample of 5 marbles is drawn from the urn. Find the probability that the sample contains exactly 1 black marbles. (Ans.  $\frac{5}{32}$ )
12. Suppose 2 bad light bulbs get mixed up with 10 good ones, and that you start testing the bulbs, one by one, until you have found both defectives. What is the probability that you will find the last defective on the 7th testing? (Ans.  $\frac{2}{15}$ )
13. If you hold 2 tickets to a lottery for which  $n$  tickets were sold and 5 prizes are to be given, what is the probability that you will win at least one prize? (Ans.  $\frac{10n-30}{n^2-n}$ )

### Conditional Probability

14. 1. Six percent of the members of a club are men. Eighty percent of the men and 75% of the women have activity tickets for all the club activities. A ticket is found and turned in to the club's lost-and-found department. What is the probability that it belongs to a woman? to a man? (Ans.  $\frac{5}{13}, \frac{8}{13}$ )
15. An experiment consists of throwing a three-sided die and then, depending on the outcome of the throw, selecting ball from one of the two urns. If the die falls "1" or "2", the ball is drawn from an urn containing 1 red balls and 4 black balls; if the die falls "3", the ball is drawn from an urn with 3 red and 2 black balls. You didn't see the die thrown, but you observed that a red ball was drawn. What is the probability that it come from the 1st urn? (Ans.  $\frac{2}{5}$ )
16. Find the probability that a poker hand of 5 cards will contain only black cards, (a) given that it contains at least 4 black cards, (b) given that it contains at least 4 spades. (Ans.  $\frac{11}{76}, \frac{37}{102}$ )
17. Each of 3 boxes has 2 drawers. One box contains a gold coin in each drawer, another contains a silver coin in each drawer, and the third contains a gold coin in one drawer is opened and found to contain a gold coin. What I the probability that the coin in the other drawer is also gold? (Ans.  $\frac{2}{3}$ )
18. A bag contains 2 black balls and 2 white balls. A ball is drawn and replaced by a ball of the opposite colour. Then another ball is drawn from the bag. Find the conditional probability that the first ball drawn was white, given that the second ball drawn was white. (Ans.  $\frac{1}{4}$ )
19. Two dice are rolled. What is the probability that the sum of the faces exceeds 8, given that one or more of the faces is a 6? (Ans.  $\frac{7}{11}$ )
20. A card is drawn from an ordinary desk. What is the probability that it is a king, given that it is a face card? (Ans.  $\frac{1}{3}$ )